Contextualising space: Using local knowledge to foster students' location and transformation skills

Danielle Harris
University of Canberra
<danielle.harris@canberra.edu.au>

Tracy Logan
University of Canberra
<tracy.logan@canberra.edu.au>

Tom Lowrie
University of Canberra
<thomas.lowrie@canberra.edu.au>

Location and Transformation skills are critical tools for navigating the world and establishing foundational steps for geometric reasoning associated with co-ordinate grids and the Cartesian plane. The contextual nature of using local landmarks to understand students' mental representation of large-scale space has the potential to enhance these skills. This paper examines a classroom activity that draws on students' local knowledge when representing their environment. Factors such as geographic distance and isolation, and incorporation of spatial relations are explored. Recommendations are made for educators to incorporate the sophisticated local knowledge when building mathematical understanding.

From the Foundation year of school, the Australian Curriculum identifies *Location and Transformation* as critical elements of mathematics (ACARA, n.d.). This content sits within the general capability of spatial reasoning. Although identified in the Australian Curriculum, educators are left with little support for incorporating spatial instruction in their teaching (Lowrie & Logan, 2018). Engaging with position and movement provides a novel opportunity to embed learning into tangible, real-world, contexts for students. Rather than abstract notions of mathematical content confined to a page or screen, teaching about large-scale space affords students the opportunity to be active participants in their learning. Physical exploration has been linked to greater accuracy and flexibility when estimating landmarks and distances compared with abstract (i.e., virtual) experience (Richardson et al., 1999). This embodied approach to spatial reasoning has been found to be effective in mathematical and cognitive learning models (Nathan et al., 2020; Tversky, 2009). To address the problem of how to bring spatial instruction into the classroom in an accessible, contextualised way, we explore engagement with a spatial task that drew on the local knowledge of students from culturally and geographically diverse schools.

Location and Transformation

Location and Transformation are interwoven throughout the Australian mathematics curriculum. In the early years, the focus is on position and movement to assist with simple directions. As students develop, they are taught increasingly complex mapping skills as a foundation for the introduction of the Cartesian coordinate system (ACARA, n.d.). Despite the inherently spatial nature of this content, concept development often fails to consider the opportunities of promoting spatial representations to provide students with a fallback strategy when content difficulty increases (Lowrie, Logan & Patahuddin, 2018).

Location is a broad term spanning Measurement and Geometry, ranging from descriptive language (i.e., *behind* or *next to*), to pictorial (grid representations), and symbolic (co-ordinate systems). This learning progression was identified by Lowrie, Logan and Patahuddin (2018) as critical for development of sound mathematical understanding. They

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posit that student experiences support language growth and engagement with pictorial representations (i.e., concrete materials, gesture, maps, pictures). It is these foundations that foster development of symbolic understanding and further applications to more complex mathematical concepts.

Large-scale Spatial Representation

Mapping skills sit at the nexus of numeracy and spatial cognition. Numeracy (via *Location and Transformation*) and Geography curriculums emphasise the development of mapping skills throughout schooling (ACARA, n.d.), while psychologists explore the relationship between mental representations of real and virtual environments to understand the development of navigation skills (Keil et al., 2020; Richardson et al., 1999).

Drawing on student experience is critical when developing mathematical and spatial thinking (Lowrie, Logan, Harris et al., 2018). Connecting new learning to students' knowledge provides the foundation for language development such as directional and relational language (e.g., the park is south of school, I go past the corner store on my way). Although language alone is not sufficient for developing spatial thinking (as this would undermine the non-verbal nature of the concept), language can be critical for directing attention and building towards more complex spatial concepts (Newcombe & Stieff, 2012). Experience and language lay the groundwork for developing increasingly sophisticated large-scale spatial representations and map understanding (Larkin & Kinny-Lewis, 2017). These tools transcend cultural boundaries and provide access points for all students when building content knowledge.

Large-scale spatial representation has traditionally been thought to reflect a cognitive map incorporating Euclidean space, landmarks, and routes (Tversky, 2003). Although cognitive maps develop through exposure to both physical space and maps, the notion that the representations themselves are map-like is a topic of some debate (Foo et al., 2005). Some researchers have argued that mental representations of large-scale space may be more like graphs, with spatial locations represented as nodes, connected by familiar routes but flexible enough to account for changes in orientation and task demands (Peer et al., 2021).

Spatial Relations

Landmarks serve two main purposes in spatial representations (Presson & Montello, 1988): 1) as navigational cues, and 2) as reference points for determining spatial relations (Clements & Battista, 1992). Here we focus on spatial relations, however the salience and organisation of landmarks in the spatial representation can be highly contextual. For example, a student may recall passing the park and shops on their journey to school, but it does not necessarily help them position the locations from a birds' eye perspective.

Scale adds an extra element to the notion of spatial relations. The structure of large-scale space is divided into regions that, even in the absence of language, can be thought of in terms of distance and direction (Kuipers, 1978). By removing physical boundaries, students are free to reveal the scale and relative position of the landmarks as they exist in their mental representation. It is through this physical enactment of their mental representation of space that we can gain insight into their awareness of their local environment, including scale and relative position, and use this as a springboard for developing further content knowledge.

The Context of the Study

Research has shown that a great deal of curriculum content is established in a city-centric style that leaves students in regional and rural communities at a disadvantage (Roberts, 2017). However, recent work has highlighted the incredibly sophisticated local knowledge possessed by students outside of city centres (Lowrie et al., 2021). It is this contextualised knowledge we propose provides curriculum accessibility for all students in developing *Location and Transformation* understanding.

When performing tasks relating to their local environment, visual prompts allow children to recall and represent a greater amount of information than free recall alone (Matthews, 1985). Therefore, by providing students with physical stimuli we can explore children's representation of space using familiar landmarks (Peer et al., 2021). Tversky and Hard (2009) argued that the mere presence of an individual in a spatial perspective task alters the interpretation of spatial relations. In this study, while all students were oriented to face north, relations between landmarks were relative to the school or position of other landmarks (as determined by the student).

This study is situated within an Australian Research Council Discovery Project exploring spatial reasoning in children from culturally and geographically diverse communities. Specifically, this study examined students' large-scale spatial representations, with a focus on factors such as geography, distance, and spatial relations, with the goal of analysing the efficacy of using local knowledge to foster foundational spatial concepts.

Method

Participants

Thirty Grade 5 students from three NSW schools participated in this study. The sites represent vastly different geographic locations and population density: an urban site in Western Sydney, a rural site (population < 1,000), and a regional site (population > 30,000).

Procedure

Students were shown a collection of local landmark sites (such as parks, shops, prominent town features) and asked whether they recognised the site. They were asked how often they visited or travelled past the site, whether they had positive or negative feelings about the location, and how familiar they were with the site.

Students were seated facing north and given a piece of A3 paper with a dot representing the school in the centre. As each site varied significantly in terms of geography and density, the school was chosen as a central point as it was familiar to all students, and consistent within and between sites. Students placed the photos of landmarks they recognised around the school point from a bird's eye perspective over their local area. Students performed this task twice on consecutive days with different landmarks. The photos were large compared to the school marker and the A3 paper. There were no constraints on the way students were able to complete the task and all photos were provided to the students at the same time.

Scoring and Analysis

We analysed student representation according to three criteria, and then made site-based comparisons using Analysis of Variance, and Nonparametric tests (chi-square) to explore distributions within sites:

1. *Landmark recognition* = proportion of the possible landmarks recognised
2. *Landmark accuracy* = landmarks positioned correctly relative to school
3. *Spatial relations* = the scale and relative position of landmarks
   a) *Scale* = some photos placed further than others
   b) *Relative position* = clustering of photos

Results

Landmark Recognition

A 3x3 mixed factorial ANOVA revealed significant main effects in landmark recognition across distance categories (within-groups) and site (between-groups), and a significant interaction, $F(4,54) = 3.85$, $p = .008$, partial eta^2 = .22. All students recognised a larger proportion of near landmarks, $F(2,26) = 19.59$, $p < .001$, partial eta^2 = .60. Between sites, rural students recognised a significantly larger proportion of landmarks than urban students, $F(2,27) = 4.13$, $p = .027$, partial eta^2 = .23. Means are presented in Table 1.

Table 1
Average percentage of sites identified in each of the distance categories

	Near (<1 km)	Intermediate (1-5 km)	Far (>5 km)	Total
Urban	56%	57%	56%	56%
Rural	94%	76%	57%*	76%
Regional	100%	50%	43%	64%

**Note.* All far landmarks in the rural site were located in neighbouring towns roughly 40-50km away.

Urban students recognised half of all landmarks across distance categories, while regional students were familiar with all locations within 1 km of school, dropping to half the sites beyond 1 km. By contrast, rural students identified a large proportion of landmarks in their own town. Despite the distance of the far landmarks, rural students still identified more than half the possible landmarks.

Landmark Accuracy

A 3x3 mixed factorial ANOVA revealed significant main effects in accuracy by distance (within-groups) and site (between-groups) and a significant interaction, $F(4,54) = 2.88$, $p = .031$, partial eta^2 = .18. Landmarks in the near range were positioned most accurately, $F(2,54) = 13.60$, $p < .001$, partial eta^2 = .34. At the school level, rural students were more accurate than urban students, $F(2,27) = 8.21$, $p = .002$, partial eta^2 = .38. At the urban site there was no difference in performance based on distance categories while rural and regional students experienced decreasing accuracy as distance increased. Regional students had a sharper decline with increasing distance than rural students (see mean percentages in Figure 1).

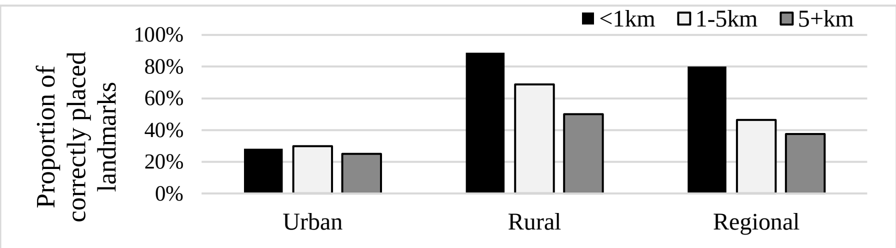
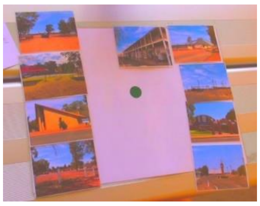


Figure 1. Mean percentage of landmarks in correct position relative to school site

Spatial Relations

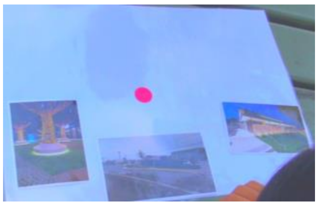
Students used different strategies to demonstrate their mental representation (examples in Table 2). We analysed the final position of the photos as not all students verbalised their thinking during the task. The most distinct differences were in orientation and structure. Some students kept all photos facing themselves while others rotated the photos to reflect how the landmark would appear when journeying from school. The structure students chose when arranging the photos varied between grid-like and relational. The relational structure accounted for the scale and relative position of landmarks, or a combination of both. These differences are discussed further in the next section.

Table 2
Representation categories

Orientation	Upright	Rotated
		
Structure	Grid-like	Relational
		

Scale and relative position. A third of all students demonstrated elements of scale and relative position, however, this had no significant connection with accuracy. One exception was for those that demonstrated scale, these students were more accurate when placing near landmarks, $F(1,29) = 6.81$, $p = .014$. There were no significant differences for the other distance categories. Table 3 includes sample arrangements of the four categories.

Table 3
Sample representations, and student numbers per category

		Scale	
		Yes	No
Relations	Yes	 N = 10	 N = 8
	No	 N = 7	 N = 5

Non-parametric analysis (cross tabulation using chi-square statistics) revealed a difference by site in the representation of scale, $\chi^2(2) = 8.69$, $p = .01$, but not relative position, $\chi^2(2) = 46$, $p = 79$. A large proportion of students in the rural and regional sites represented scale compared with only one urban student. Despite the distance category parameters remaining constant, students at the urban site appeared less sensitive to the distance when arranging the landmark photos.

Discussion

Recognition and Accuracy

Landmarks within 1 km were most recognisable and positioned with the most accuracy (with the exception of urban students). Regional students were incredibly familiar with their local area, within 1 km, recognising all the possible landmarks and accurately positioning 80% of those. More progressive, rural students were able to recognise most landmarks within their town and still more than half of the landmarks in towns 40-50 km away. Even at this distance, rural students correctly placed roughly half of the landmarks, which was more than the urban or regional students whose far landmarks were roughly 5 km away. Tversky (2003) talks about key landmarks when referring to cognitive maps. In towns like the rural one in this study, the geographic size and relatively low density may contribute to students being aware of all landmarks. By contrast, the density of the urban environment makes competition for landmark memory much higher. For example, most students in the regional town were able to identify something as routine as a street sign, while at the urban site only a McDonald’s and a movie theatre were consistently recognised. The regional students similarly recognised a local McDonald’s but were also able to identify local parks, shopping centres, petrol stations and hardware stores. It is possible there are fewer of these to compete for attention, or the nature of children’s lived experience drives their memory for these locations. This finding has implications for classroom practice, the richness of local knowledge demonstrated by rural and regional students can be drawn upon when introducing concepts such as scale. When verbalising their thinking, those students who demonstrated scale and relative position were able to clearly articulate the relations between the sites, and often drew on these relationships to help them position less familiar photos.

Presson and Montello (1988) discuss the importance of context when it comes to spatial memory for location. Our results highlight the impact of student context in mental and physical representation of their local environment. We argue that the sophisticated local knowledge in rural and regional areas should be harnessed when building understanding around *Location and Transformation*. Similarly, it would benefit urban students to engage more with their local environment, for example through community walks or mapping exercises, to provide foundational experiences and develop directional language before building towards more abstract representations of space. Educators are well-placed to draw on student strengths and experiences when building mathematical knowledge – this task is one example of how local knowledge can be used.

Spatial Relations

The open-ended nature of the task allowed students to reveal the diversity of their mental representations of the local environment. While some students kept all photos upright, others rotated the images to align with their view as they mentally traversed the journey. This latter approach may be indicative of the graph approach (Peer et al., 2021), with students connecting nodes (i.e., locations/landmarks) via their well-travelled routes. Anecdotal evidence from some students' reflections suggested that these differences may be due to map-like (i.e., bird's-eye) versus route-based strategies. Future research may benefit from exploring these distinctions further.

Relative position. Despite the body of work discussing relative position as a critical component of the accuracy of spatial representations (Peer et al., 2021; Presson & Montello, 1988), our findings did not establish a link between students who demonstrated relative position and their accuracy in positioning landmarks. The difference between our study and those before are that we drew on the local environment in selecting landmarks, whereas previous studies have focused on new learning. In these instances, the locations (or nodes) under consideration are determined by the researcher. In our study it may be that students were drawing on knowledge beyond what we presented to them, for example a third site (such as home) may have helped them triangulate locations (Foo et al., 2005).

Scale. By contrast, representation of scale did show significant connections to accuracy and context. Those that demonstrated scale by positioning the photos at varying distances from school were more accurate in their placement of near landmarks. It is one possibility that these students had a robust mental representation of their local area and then used this to extrapolate to the larger area. In newly learned environments nearer landmarks have been shown to be associated with greater salience and accuracy (Keil et al., 2020).

Consistent with the notion that context is critical when examining *Location*, rural and regional students were more likely to represent scale. The nature of their interaction with their local area appears to have a bearing on their awareness of the scale of the environment. Many rural students travel long distances by bus to school while many regional students reported not travelling very far beyond their local community in their daily lives. Both environmental conditions may contribute to students' sense of environmental scale (Presson & Montello, 1988). Scale and magnitude are foundational numeracy skills, our findings suggest that where city-centric teaching models may disadvantage some students (Roberts, 2017), the opportunity to draw on students' local knowledge and experience may make abstract mathematical concepts more accessible for all students.

Future Directions

This task provided some insights into the different ways students represent large-scale space. The factors explored in this paper were broad in terms of geography and assumptions about student experience of both the sites and town structure. Future research may look at more individual factors, such as students' freedom to roam, means of transport, and family culture. Although we explored the use of relative position and we did not analyse the order in which students placed the photos, it is possible more in-depth analysis of the students' actions and thinking may give insights into key landmarks (or nodes) around which their spatial representations were built.

Conclusion

Much of the spatial research examines lab-based or abstract notions of spatial reasoning which often leaves students in regional and rural areas at a disadvantage. We have visited sites with different social, geographical and cultural contexts. We have chosen to examine the question of spatial representations with a different lens. Our results indicate that the engagement with the local environment afforded by rural and regional living has provided students with an advantage in representing their familiar space. We suggest that this embodied, contextualised spatial knowledge is a strong foundation for building mathematical knowledge around *Location and Transformation* as a springboard for more complex mathematical skills.

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